CALCULUS STUDENTS' SOURCES OF CONVICTION*

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Research findings from extensive task-based interviews with introductory calculus students are discussed. The research was conducted from a constructivist perspective. Students' sources of conviction were the focus of analysis, with sources of conviction referring to how one determines mathematical truth and validity. The existence and characteristics of three groups of calculus learners were revealed: Collectors, Technicians, and Connectors. The groups differed in the nature and role of their sources of conviction and manner of construction of calculus conceptualizations.

Few studies have been done to examine student learning in calculus and many that have been done have focused on student errors, misconceptions or inability to perform certain tasks (for example, Davis and Vinner, 1986; Orton, 1983a, 1983b; Seldon et al. 1989). These investigations have given insight into students' misunderstandings in calculus. What is needed now is research into how instruction can better guide and support student learning in calculus. This is particularly important when one considers that the drop-out and failure rates in calculus are high compared to other undergraduate courses. Figures between 30% and 50% are reported in the literature (Peterson, 1987; Cipra, 1988). Even students passing a calculus course tend to perform at low levels with respect to both skills and the use of calculus ideas (Peterson, 1987; Cipra, 1988).

For improved student learning in calculus more research is needed into various instructional emphases and formats and their subsequent effects on learning. One perspective by which such research might be formulated is that of constructivism. Constructivism has emerged as an important influence in mathematics education research. This is because constructivism has provided a valuable perspective from which to understand mathematics learning (Ernest, 1989). A constructivist model of knowledge views mathematics learning as an individual, evolutionary process (Kilpatrick, 1987; von Glasersfeld, 1984, 1987). This is in contrast to a view of learning that sees concepts as transferable, "ready-made" from teachers to learners. That is, constructivism views mathematics learning as an active, constructive process in which individuals build up knowledge for themselves.

Ernest (1991) discusses the above notions through discussion of mathematics as a social construction. This philosophy, known as social constructivism, takes the view that "human language, rules and agreement play a key role in establishing and justifying the truths of mathematics" (p.42). Mathematical knowledge is thereby seen to be grounded in the following: (1) "linguistic knowledge, conventions and rules" (p.42), (2) social processes by which an individual's internal, subjective knowledge is turned into external, objective knowledge, and (3) objectivity viewed as public, social acceptance rather than an inherent property of the content of knowledge. These features imply that mathematical knowledge is dependent upon social sharing of language and decisions pertaining to truth and validity. Further, as a consequence for mathematics education researchers, these points imply that ways of determining truth and validity are likely to be important components of mathematics learning.

This is a report of a portion of a study of student learning in calculus from a constructivist perspective. The parts of the investigation presented here focus on the nature and role of students' convictions regarding the validity or truth of calculus interpretations and problem responses and the ways students construct their calculus conceptualizations. The term *sources of conviction* is used to refer to how one determines mathematical truth and validity, or more specifically, how one determines facts, accordance with accepted mathematical principles and standards, legitimacy, consistency and logicality. Responsibility for the determination of truth or validity could lie within various sources, including the teacher's knowledge, the statements of a textbook or other instructional materials, the inherent physical structure of the world, a student's knowledge of the structure and rules of mathematics, or a student's own personal beliefs.

METHOD

The research was a naturalistic study involving three undergraduate calculus classes located at three different post-secondary institutions in Western Canada. Included were a large university and two small private colleges. Task-based and personal interviews with 17 students were the method of inquiry into the nature and role of students' *sources of conviction* and manner of construction of calculus conceptualizations. The interviews were conducted in the last three weeks of a 13 week school term, and each lasted one to two hours. A twenty to thirty minute follow-up interview was conducted 2 to 3 weeks later to have a student clarify or expand upon responses from the first interview. The problems given to students asked them to identify, describe, interpret, explain or apply limit and derivative concepts and they included open-ended as well as relatively focused tasks. The initial written problems were standardized (i.e. the same for all students) and subsequent written and oral questions asked by the researcher were contingent upon a student's previous responses. The interviews also incorporated relevant personal questions related to students' perceptions of calculus and the learning of calculus, study practices, ways of determining "correctness" and attitudes towards calculus. The overall manner of proceeding with analysis of the interview transcripts was similar to that used by Belenky et al. (1986) to analyze interviews conducted "to explore with women their experience and problems as learners and knowers" (p.11).

RESULTS

Student interview data revealed the existence of three groups of students who differed in their *sources of conviction*. These groups were named Collectors, Technicians, and Connectors. The names reflect the nature and role of the groups' *sources of conviction*. For the 17 interview students, 8 were classified as Collectors, 4 as Technicians and 5 as Connectors. Each of the three groups are discussed in upcoming sections. The groups differ from each other in the degree to which their *sources of conviction* are external or internal in nature. Collectors exhibit the highest degree of externalized *sources of conviction*, while Connectors exhibit the highest degree of internalized *sources of conviction*. Technicians fall somewhere in between these two other groups, exhibiting a mixture of external and internal *sources of conviction* .

Collectors

Students who from their *sources of conviction* are classified as Collectors display *sources of conviction* that are generally external in nature. These *sources of conviction* are external in that they reside in statements, rules and procedures presented by the teacher or textbook. They do not generally reside in what students have construed for themselves. The students construct their mathematical knowledge by assembling isolated, relatively unconnected mathematical statements, rules and procedures. Thus, a Collector's calculus conceptualizations can be said to be a "collection" of statements, rules and procedures. More specifically, the external nature of a Collector student's *sources of conviction* guides the student to approach calculus learning as recall or rote memorization of statements, rules and procedures. In this way the role of a Collector's *sources of conviction* is as a validation to the student that he or she makes statements and performs procedures that will be recognized as valid or correct by other individuals. Although the student might validly apply calculus knowledge, the student does not claim to know personally whether particular pieces of mathematics are valid or correct. Rather, the student relies on others to determine validity or correctness. These other individuals are perceived by the student to be people for whom calculus is understandable and meaningful.

A distinctive feature of Collectors' *sources of conviction* was their external nature. They said such things as:

Doug: Like I could say I remember in class that if you have this situation there's no derivative, but I don't know why there's no derivative.

Ellen: I don't know if this is right, but I think I remember something like that from the textbook.

Gordon: I don't know why. I just remembered something about there's not a derivative.

In all the above interview extracts the students make reference to what they remember from their class or textbook, using the teacher or the textbook as a *source of conviction.* These *sources of conviction* are external in nature in that the students employ them to reproduce what they remember from class or the textbook. More explicitly, the students use the teacher or textbook as a means of validation, while they concurrently state they "don't know" (Doug) why a particular piece of mathematics is as it is. That is, the students do not claim any ownership of the calculus concepts, rules or procedures they use.

Another feature of Collectors' interviews is that Collectors were often unsuccessful in completing the interview problems. Collectors frequently made errors, displayed misconceptions, were unable to remember particular rules or procedures or were unable to explain concepts. For example, they frequently did not completely or correctly remember such things as the product rule, the quotient rule or the chain rule. They also displayed a lack of ability to explain derivative or limit concepts.

Collectors displayed beliefs that mathematics is a collection of definite, correct formulas, rules and procedures. They expressed views that mathematics is "black and white" (Doug) and has "definite" answers (Betty) and "correct" (Daniel) ways that problems are to be solved. These views further reveal the external nature of Collector students' *sources of conviction* in that they show how Collectors perceive truth and validity decisions in mathematics as pre-determined, external entities. They do not generally see it possible that these decisions might be influenced by one's own perceptions or interpretations. Rather, they must be remembered or memorized. In fact, Collectors explicitly state they approach their calculus learning by memorization of what they believe will be needed to pass an exam. The following interview extracts provide evidence of how Collectors view their calculus learning as memorization:

Gordon: Yeah. I didn't fully understand it. I was just memorizing. Like memorizing what to do with the formulas, but not understanding why you have to do it. Like if you get the slope, like the chain law, I know what to do to find the derivative. But I don't understand how that works out.

Doug: Well I just plain don't understand calculus. Like I get the questions, but it's not because I understand them. It's because I just memorized them.

Cindy: I can kind of work out formulas and work out a way, kind of memorize almost a way that he tells us to do it. But I don't really understand it. I can understand, like I can memorize like derivatives.

In the above excerpts the students state they use memorization as a learning technique in calculus. They speak of "just memorized" (Doug) formulas and examples and they make it clear they do not feel they personally understand calculus in terms of why one uses particular procedures or how procedures function in reaching a solution.

From the discussion thus far it can be concluded that Collectors' externally oriented *sources of conviction* do not promote a sense of personal understanding or ownership of calculus skills and conceptualizations. In fact, Collectors often spoke of calculus as being separated from their reality and ways of understanding. Comments on their impressions of calculus included:

Cindy: Well I might be able to write it down, but it probably wouldn't be right. I probably wouldn't do it the correct way. But I would, if I was to go back and read it I would understand what I meant. But it wouldn't be the right way so anybody else would understand it.

Doug: It just doesn't come to me easily. So I have to really work at it. Whereas something like English I can just do it. Political science ... There I can actually use, like I can just do it with my own mind. I can give my own interpretations of something. But in math it's either right or wrong.

Ned: ... like with me I have to look through someone else's eyes. Like a foreign kind of viewpoint. That's very hard for me to do.

These excerpts demonstrate that Cindy, Doug and Ned view mathematics as a "foreign kind of viewpoint" (Ned) from which one does not use one's "own mind" or "own interpretations" (Doug). These students do not allow internal *sources of conviction* to play a prominent role in the building of their calculus conceptualizations. In particular, they do not see as valid their personal ways of interpreting or expressing mathematics. This devaluing of one's own mathematical interpretations is particularly clear in the following comments made by Daniel:

Daniel: And right now it is a matter of being able to produce it on a test. And whether or not my interpretation is correct doesn't matter. Because my interpretation isn't going to be counted on the test.

Well in most anything else I could feel confident my views are um maybe not necessarily correct, but that they're feasible, or that I can show how my views and somebody else's views correlate or something. Like you know. In math I don't feel that I have got any basis to say that I'm right and I'm wrong. Because if they, they referring to math people, come up with all this stuff, or how do I say it. I'm just not confident that my way of viewing it, like I could so easily be wrong. Like I just don't feel I have it.

At this point it must be noted that Daniel's feeling of a lack of confidence in his calculus abilities was not an isolated feeling amongst Collectors. All the Collectors except Betty and Ned explicitly expressed a lack of confidence in their abilities to personally understand calculus.

In summary, Collectors generally display *sources of conviction* that are external in nature. Their *sources of conviction* reside predominantly in statements, rules and procedures presented by a teacher or textbook. The role of these *sources of conviction* is as a validation to the student that his or her calculus statements and problem solutions will be recognized as valid or correct by mathematicians or mathematics teachers. By way of this role Collectors' calculus conceptualizations are constructed as an assemblage or collection of relatively unconnected mathematical statements, rules and procedures. Collectors generally believe mathematics has definite rules and procedures and is a dichotomy of right and wrong problem solutions. They generally speak of mathematics as separate from their own reality and they sometimes explicitly devalue their personal interpretations of calculus. In addition, most of the Collectors explicitly expressed a lack of confidence in their calculus abilities.

Technicians

Technicians display a mixture of internal and external *sources of conviction*. Their external *sources of conviction* are similar to Collectors' in that they are based on knowledge of calculus statements, rules and procedures. However, Technicians differ from Collectors in their perceptions and use of these statements, rules and procedures. Technicians see calculus as a logical organization of statements, rules and procedures and they employ this organization as a technique for thinking about and applying calculus concepts. What therefore most distinguishes Technicians from Collectors is that Technicians display personal knowledge of how calculus statements, rules and procedures fit together into a logical whole. This logical whole thereby becomes a calculus "technology" in that it is a science or method for thinking about and applying calculus. Technicians can therefore be viewed as skilled users of the application of calculus techniques. Thus, the role of a Technician's *sources of conviction* is as a set of tools that the technician employs to apply calculus concepts.

However, although Technicians make use of calculus statements, rules and procedures as external *sources of conviction*, a complete examination of their interviews reveals their calculus conceptualizations are more organized than Collectors'. Their knowledge of calculus statements, rules and procedures is more than the "collections" displayed by Collector students. More specifically, instead of a collection of relatively unconnected mathematical statements, rules and procedures, Technicians' *sources of conviction* are based upon statements, rules and procedures organized into a coherent, structured set. This set is then employed as a logical technique to think about and apply calculus concepts. The existence of structured calculus conceptualizations and related *sources of conviction*, rather than an unorganized "collection", is partially evidenced by the fact Technicians generally displayed more extensive calculus knowledge and skills than Collectors. An example of a technician's perceptions of calculus as an organized structure are displayed in the following extracts from the interview with Jennifer:

Jennifer: The derivative. I don't know. It seems like it is a base that you can work from. And everything seems to rely on it. Like plugging values back into it. The points of inflection and critical points and stuff like that.

Understand something? To take that tiny basis of logic and be able to build on it. Like using that maybe as a cornerstone. But if you understand that, then you can understand things more ... Then you can continue onto a higher level ... By applying to another concept. How can I say it? Through practical application.

I think calculus, if you get into a method of thinking it's just a process. It seems to be the same sort of process and you just get into that method of thinking and it's all very logical.

In these interview extracts Jennifer speaks of calculus as a building and application of a structure of statements, rules and procedures. She speaks of knowing how to apply calculus ideas to solve a problem. It is this sense of knowledge of how calculus is structured as an applicable technology that most distinguishes Technicians from Collectors. For example, Jennifer is a Technician rather than a Collector in that she sees calculus ideas and procedures as something one can "build from" and employ as a "process" or "method of thinking" to "work through" and solve problems. These perceptions of calculus as a "method of thinking" (Jennifer), a "pattern" (Richard) or a logical "step by step" (Sally) problem solving process were not present in Collectors' interviews.

Thus, a prominent aspect of Technicians' *sources of conviction* is they are based upon knowledge of calculus as an applicable process or technology for solving problems. What is not clear thus far is whether or not *sources of conviction* that reside in the technology of calculus are internal or external in nature. When one examines comments on Technicians' own calculus knowledge, a mixture of external and internal *sources of conviction* appear. As an example of this aspect of their *sources of conviction* Sally's impressions of and experiences in calculus will be outlined. Sally spoke of both memorizing rules and working through calculus problems for herself. She said such things as:

Sally: Understanding is applying the ideas to get a right answer ... And you need to know the ideas in order to apply them. And know what ideas apply in what circumstances.

When I do a question and I look at the book and the answers match. I guess that the whole way of understanding calculus for me is getting it right. But I guess for a lot of people, you know, it shouldn't be that. It should be just knowing the ideas. But for me it isn't.

I'm trying to get the ideas into my head and how to use those ideas. Like the examples especially. I follow them closely, step by step, what he's doing, how he's applying the ideas to a problem. And then I try to do them myself. And the exercises from the book. And then also, I guess it's just memorizing rules. Though that's not what math should be. But it is a bit because you have to remember the rules in order to use them.

I: Do you feel a need to convince yourself?

S: Yeah. Which is why I study. Why I do questions and assignments. To make sure.

Sally speaks of her calculus learning in terms that reflect both external and internal *sources of conviction*. The external nature of her *sources of conviction* are seen in her references to "just memorizing rules" and getting answers that "match" the textbook answers. Her internal *sources of conviction* are seen in the fact she tries to do problems and exercises for herself, trying to learn "how to use" the ideas. Sally speaks of calculus as learning "ideas in order to apply them." To her, calculus is a technology of knowing "what ideas apply in what circumstances." Although she does problems for herself to "make sure" she is convinced of the validity or correctness of her work, she sees calculus understanding as getting the right answers. In other words, Sally's *sources of conviction* simultaneously reside in externally oriented statements of the textbook and personal, internal knowledge of the application of ideas. This display of a mixture of external and internal *sources of conviction* was also evident in the interviews with the other three Technicians, Jennifer, Nadine and Richard.

In summary, Technicians generally display a combination of external and internal *sources of conviction*. Their external *sources of conviction* reside in knowledge of calculus statements, rules and procedures, while their internal *sources of conviction* arise from knowledge of how to use these rules and procedures. Thus, the role of a Technician's *sources of conviction* is as a means to organize and structure calculus statements, rules and procedures. The resultant structures thereby become a calculus technology that guides and informs a student in the application of calculus ideas and techniques. It is through a sense of mastery of the technology of calculus that Technicians' *sources of conviction* are more internal in nature than Collectors'.

Connectors

Students who from their *sources of conviction* are classified as Connectors display *sources of conviction* that are generally internal in nature. These *sources of conviction* are internal in that Connectors display a sense of being able to interpret calculus for themselves. Similarly to Technicians, Connectors display knowledge of calculus as a technology. They organize their calculus experiences so as to be able to logically and consistently apply calculus ideas and techniques. However, Connectors differ from Technicians in that they display a stronger sense of personal understanding of their calculus conceptualizations. They also display a higher degree of competence in both explanation and application of calculus. Their conceptualizations are displayed as a network of "connections" between various aspects of calculus and between calculus and themselves. In this way the role of a Connector's *sources of conviction* is as a validation to the student that he or she makes statements, performs procedures or creates problem responses that are valid, correct and meaningful to the student as well as other individuals. Thus, Connectors are able to both apply calculus knowledge and make personal sense of this knowledge.

Connectors frequently spoke of approaching their learning as trying to understand for themselves and trying to connect together ideas, statements, rules and procedures. Examples of what they said in relation to these two features are:

Annabel: I'm trying to fit it all together. Not memorizing.

Neil: I definitely try to recreate things and think it through ... Seeing how everything is linked together. And not just this idea, and this idea over here. And if they are connected then one should know it. Even if it's a little more complex. But, I think the connections are important.

Mike: I like, like I said, I like to know how it works for myself. And figure things out for myself. You have more control that way.

Tanya: Because you can't learn from memorizing everything. Because you have to interpret it. You have to understand the theory behind a certain form. The theory behind a certain something, and then apply it to something else. ... Cause you need to, you need to imagine it in your head. What goes on. You can't, you can't see infinity. You have to imagine infinity. You can't see infinitely, or infinitesimally small. You have to imagine it.

In these excerpts the students speak of understanding rather than memorizing calculus. In comparison, none of the Collectors said calculus made sense to them. Technicians expressed some sense of understanding of calculus, but they did not speak of their learning of calculus in the same way as did Connectors. Connectors spoke more of calculus as something one learns through personal involvement with and subsequent flexible application of ideas. This aspect of their learning is particularly clear in Tanya's comments on her learning. She speaks of her "imagination" as an essential component of her calculus learning and notes how she must "interpret" rather than memorize in order to learn how to apply calculus theory. Through these words Tanya expresses a sense of personal understanding or ownership of her calculus knowledge. That is, as *sources of conviction* she uses knowledge and thought processes that she conceives of as her own. This sense of oneself and one's own thought processes and interaction with material as *sources of conviction* by which to learn and use calculus is also seen in the other Connectors' words.

The sense of personal control and involvement of one's own thought processes that is demonstrated by Connectors reveals the internal nature of their *sources of conviction*. More specifically, Connectors' *sources of conviction* are internal in nature in that they reside in a sense of personal comprehension and control of calculus ideas and applications. In this way, the role of Connectors' *sources of conviction* is as both a guide and a confirmation for students that they state and use calculus ideas and applications in ways meaningful to themselves as well as others knowledgeable in calculus.

Another prominent feature of Connectors' interviews was they displayed a higher level of competence with calculus concepts and skills than the other interview students. This fact points to a relationship between high competency in calculus and approaching calculus learning as a Connector, but it is not clear if one causes the other. Connectors' problem responses were often more detailed than the other students', using more symbolic representations and more complete explanations of ideas or procedures. In comparison to the problem responses given by Collectors and Technicians, Connectors' problem responses showed more facility with calculus ideas and techniques.

In summary, Connectors generally display *sources of conviction* that are internal in nature. Their *sources of conviction* reside largely in ideas and techniques they perceive to make sense. That is, Connectors view calculus knowledge as something of which they can gain personal understanding and use. They speak of approaching their calculus learning in terms of aiming to understand, make sense of and flexibly think through and apply ideas and techniques. In this way Connectors use their internal *sources of conviction* to construct calculus conceptualizations of which they feel personal understanding. The role of a Connector's *sources of conviction* is therefore as a guide and a confirmation to the student that she or he makes statements and performs procedures that are meaningful and useful to the student as well as other individuals. Connectors see their own interpretations and thought processes as components of their calculus learning. Their calculus conceptualizations are thereby constructed as a network of personally meaningful, interconnected statements, rules and procedures.

SUMMARY, DISCUSSION AND IMPLICATIONS

Student interview data revealed the existence of three groups of students who differed in their *sources of conviction*. These groups were named Collectors, Technicians, and Connectors. Collectors exhibited the highest degree of external *sources of conviction*, using teacher or textbook presentations as means by which to determine truth or validity. Their calculus conceptualizations were constructed as a collection of isolated, relatively unconnected statements, rules and procedures. Technicians based truth and validity upon their knowledge of the logical, organized structure of calculus and constructed their conceptualizations as a logical organization of statements, rules and procedures. Connectors exhibited the highest degree of internal *sources of conviction*, displaying a sense of personal understanding of calculus. Their conceptualizations were constructed as a network of connections between various aspects of calculus and between calculus and themselves.

Since few of the students in this study were Connectors, it is apparent that the search for effective ways to guide students to personal understandings of calculus must continue. Regardless of whether or not students apply calculus or study calculus beyond an introductory level, it is desirable that they pursue their calculus learning as a meaningful endeavour. What is noteworthy here is that students who saw their calculus learning as personally understandable displayed more competence, confidence and satisfaction in their abilities to do calculus. Technicians used knowledge of calculus language as a technology by which to apply calculus, but their related conceptualizations were not necessarily perceived by them to be personally meaningful. In other words, mastery of the use of calculus language can help students attain competence with calculus skills and basic ideas, but it does not necessarily guide them to personal understandings of calculus conceptualizations. Thus, it appears that language use is an important vehicle by which calculus students might be better guided to calculus learning as a meaningful endeavour.

Also of particular note is the fact that over half the interview students were classified as Collectors. These Collectors, although engaged in conceptual constructions, did not claim any personal understanding of their calculus conceptualizations. The fact that they conceived of calculus learning as replication of teacher or textbook presentations implies calculus instruction might be more successful for them if methods were developed that encourage them to take more personal involvement in the construction of their calculus conceptualizations.

Two other features which emerged during analysis of student interviews are worthy of further investigation aimed at clarification, refinement and generalizability of notions. These features are:

(1) A primary area of examination in this study was students' *sources of conviction.* This concept raises a number of issues in need of further research. First, studies should be undertaken to investigate whether the three groups of students, Collectors, Technicians and Connectors, are present in other groups of calculus students. Whether these groups are present in students studying mathematics at other levels or studying other subjects also needs to be determined. Such studies would contribute to the generalizability of this study and would aid further application of constructivism to mathematics education and other areas of education.

(2) Research needs to be done to determine if the nature of Collectors', Technicians' and Connectors' approaches to calculus learning form a series of transitional learning phases. For example, it is not known if being a Technician might be a transitional phase between being a Collector and being a Connector.

Given the above points, it would be advantageous to continue research into mathematics students' *sources of conviction.* Researchers and teachers would then be better guided in the development of mathematics instruction that facilitates meaningful mathematics learning.

Postscript: For a more complete report of the research on which this paper is based see Frid (1992) or contact the author.

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